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On the Issue of the Born-Rytov Controversy:

I. Comparing Analytical and Approximate Expressions for the One-dimensional Deterministic Case

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Abstract

The applicability and domains of validity for the Born and the Rytov methods in scattering theory are established, with mathematical rigor, by comparing successive terms of the Born and the Rytov series calculated for wave propagation in a homogeneous dielectric half-space and a homogeneous dielectric slab, for which the permittivities are assumed to have a low contrast over that of free-space. While the (first-order) Rytov approximation is superior to the (first-order) Born approximation when it is applied to estimate the scattered field in the homogeneous dielectric half-space, both approximations are inapplicable to estimate the scattered fields when the dielectric slab is very thick.

1. Introduction

There is much interest in modelling the propagation and scattering of waves in inhomogeneous media, to solve both direct and inverse scattering problems. The physical properties of these media can be extracted or predicted from the measurable quantities such as the scattered fields and intensities which can be derived from the models. Usually, approximations are sought in order to make the scattered fields (which can be expressed as the Fredholm integral equations of the first kind) more tractable for numerical computation. Two first-order approximations (i.e., the Born and the Rytov approximations) are most commonly exploited in the Born and the Rytov methods. However, the application of the Rytov method [1-68] has raised considerable debate over its relative merit and applicability over the Born method and vice-versa [46]. For a scatterer of compact support (with d as the size of its largest dimension), the empirical remark is usually made that the Born approximation is valid only when the scatterer is small on the scale of the incident wavelength λ in free-space and the strength of its scattering function V is weak, i.e., low contrast with respect to free-space. This can be summarized by requiring that $Vk_0d \ll 1$ where $k_0 = 2\pi/\lambda$. On the other hand, the criterion for the Rytov approximation to be applicable requires that the fluctuations in V be slow on the scale of λ but the strength of V is not necessarily small or low contrast. In other words, this physical interpretation of the validity of the Rytov approximation is based on the requirement that the absolute value of the rate of change of the complex phase of the field is very small compared with $k_0\sqrt{V}$.

In order to refine and clarify the domains of validity for both methods and thus establish, more precisely, the situations for which method is preferable over the other, we consider two deterministic scattering cases in this paper. The

first scattering geometry, described in section 2, consists of free space to one side and a dielectric half-space of permittvity $\varepsilon_0(1 + V)$ on the other. We model the propagation of the field across the interface between the free-space and the dielectric half-space and, then, make the comparison on the results obtained from the Born and the Rytov methods. In section 3, we present and compare the results derived from both methods for the second scattering geometry which consists of a dielectric slab of finite extent in the propagation direction. Based on these comparisons, we are able to delineate the appropriate domains of validity for both methods in section 4 and provide concrete conclusions, which can be generalized to other scatterers, in particular, scatterers of compact support.

2. One-dimensional Homogeneous Dielectric Half-space

Consider a time-harmonic plane wave $e^{i(k_0 z - \omega t)}$ with radial frequency ω incident from free-space (region 0 for $z < 0$) into the one-dimensional (1-D) homogeneous dielectric half-space (region 1 for $z \geq 0$) [see Figure 1]. The wavenumber in free-space is k_0 ($= \omega\sqrt{\mu_0 \varepsilon_0}$) where μ_0 and ε_0 are the permeability and the permittivity for free-space, respectively. Assume that the dielectric half-space has the permeability μ_1 and the permittivity $\varepsilon_0(1 + V)$ where the scattering function V is real and positive.

2.1. Exact Solutions derived from the Boundary Conditions

The close-form solutions for the total fields $\Psi_0(z)$ and $\Psi_1(z)$ in free-space (region 0) and in the dielectric half-space (region 1) can be easily derived from the boundary conditions:

$$\Psi_0(0) = \Psi_1(0) \tag{1}$$

and

$$\frac{1}{\mu_0}\frac{d\Psi_0(z)}{dz}\bigg|_{z=0} = \frac{1}{\mu_1}\frac{d\Psi_1(z)}{dz}\bigg|_{z=0}. \tag{2}$$

These boundary conditions can be applied for the acoustic case or the TE case in the electromagnetic wave theory. For $\mu_1 = \mu_0$, the wavenumber in the dielectric half-space becomes $k_1 = k_0\sqrt{1+V}$. After matching the boundary conditions, we obtain

$$\Psi_0(z) = e^{ik_0 z} + R(k_1)\, e^{-ik_0 z}; \qquad z < 0 \tag{3}$$

and

$$\Psi_1(z) = T(k_1)\, e^{ik_1 z}; \qquad z \geq 0. \tag{4}$$

where the reflection and the transmission coefficients $R(k_1)$ and $T(k_1)$ are given below:

$$R(k_1) = \frac{k_0 - k_1}{k_0 + k_1} \tag{5}$$

and

$$T(k_1) = \frac{2k_0}{k_0 + k_1}. \tag{6}$$

When $V \ll 1$, we can expand $\Psi_0(z)$ and $\Psi_1(z)$ into Taylor series in terms of V, i.e.,

$$\Psi_0(z) \overset{V \ll 1}{\approx} e^{ik_0 z} + \left(-\frac{V}{4} + \frac{V^2}{8} - \frac{5V^3}{64} + \ldots\right) e^{-ik_0 z}; \qquad z < 0 \tag{7}$$

and

$$\Psi_1(z) \overset{V \ll 1}{\approx} \left(1 - \frac{V}{4} + \frac{V^2}{8} - \frac{5V^3}{16} + \ldots\right) e^{ik_0 z}\, e^{i\left(\frac{V}{2} - \frac{V^2}{8} + \frac{5V^3}{16} \ldots\right)k_0 z}$$

$$\overset{V \ll 1}{\approx} \left(1 - \frac{V}{4} + i\frac{V}{2}k_0 z + \frac{V^2}{8} - i\frac{V^2}{4}k_0 z - \frac{V^2}{8}k_0^2 z^2 - \frac{5V^3}{16} + i\frac{5V^3}{32}k_0 z\right.$$

$$+\frac{3V^3}{32} k_0^2 z^2 - i \frac{V^3}{48} k_0^3 z^3 + ...\Big) e^{ik_0 z}; \qquad z \geq 0. \quad (8)$$

The above two power series are exactly equal to the Born (or the Neumann) series which can also be derived in scattering theory under the Born method, shown below.

2.2. The Born Method for the 1-D Dielectric Half-space Case

In scattering theory, $\Psi_0(z)$ and $\Psi_1(z)$ satisfy the scalar wave equations

$$\frac{d^2\Psi_0(z)}{dz^2} + k_0^2 \Psi_0(z) = 0; \qquad z < 0 \quad (9)$$

and

$$\frac{d^2\Psi_1(z)}{dz^2} + k_1^2 \Psi_1(z) = 0; \qquad z \geq 0. \quad (10)$$

Since k_1 is equal to $k_0\sqrt{1 + V}$, Eq. (10) can be rewritten as

$$\frac{d^2\Psi_1(z)}{dz^2} + k_0^2 \Psi_1(z) = - k_0^2 V \Psi_1(z); \qquad z \geq 0. \quad (11)$$

where the term on the right-hand side of Eq. (11) is the so-called secondary source due to the permittivity contrast-$\varepsilon_0 V$ of the dielectric half-space relative to that of free-space. Using the 1-D unbounded scalar Green's function $G(z, z')$ which is governed by the equation

$$\frac{d^2G(z, z')}{dz^2} + k_0^2 G(z, z') = \delta(z - z'), \quad (12)$$

we can express $\Psi_0(z)$ and $\Psi_1(z)$ as an inhomogeneous Fredholm integral equations of the first kind, namely,

$$\Psi_0(z) = \Psi_0^{(0)}(z) - k_0^2 \int_0^\infty dz' G(z, z') V \Psi_1(z'); \qquad z < 0 \qquad (13)$$

and

$$\Psi_1(z) = \Psi_1^{(0)}(z) - k_0^2 \int_0^\infty dz' G(z, z') V \Psi_1(z'); \qquad z \geq 0 \qquad (14)$$

where the unperturbed fields $\Psi_0^{(0)}(z)$ and $\Psi_1^{(0)}(z)$ are both equal to $e^{ik_0 z}$. By substituting $\Psi_1(z)$ of Eq. (4) into the integrals of Eqs. (13) and (14) and using the 1-D unbounded scalar Green's function [69],

$$G(z, z') = - \frac{i}{2k_0} e^{ik_0|z - z'|} \qquad (15)$$

it can be shown that $\Psi_0(z)$ and $\Psi_1(z)$ derived from integral representations of Eqs. (13) and (14) are consistent with that of Eqs. (3) and (4) derived from boundary conditions of Eqs. (1) and (2). Note that, in Eq. (14), the integral with limits (0 to ∞) is divided into two integrals with limits (0 to z) and (z to ∞) corresponding to two regions ($z' < z$) and ($z' > z$) as indicated by the absolute value of $|z - z'|$ in Eq. (15). The principle of limiting absorption is used for all derivations to discard the oscillating terms of the form $e^{ik_0 z}$ when $z \to \infty$ [25,41]. The Born series of Eqs. (7) and (8) in terms of the small parameter V can be derived from the perturbation method [2,6]:

$$\begin{aligned}
\Psi_0(z) \overset{V \ll 1}{\approx} \; & \Psi_0^{(0)}(z) - k_0^2 \int_0^\infty dz' G(z,z') V \Psi_1^{(0)}(z') \\
& + k_0^4 \int_0^\infty dz' G(z,z') V \int_0^\infty dz'' G(z',z'') V \Psi_1^{(0)}(z'') \\
& - k_0^6 \int_0^\infty dz' G(z,z') V \int_0^\infty dz'' G(z',z'') V \int_0^\infty dz''' G(z'',z''') V \Psi_1^{(0)}(z''') + \dots \\
\equiv \; & \Psi_0^{(0)}(z) + \Psi_{0s}^{1BA}(z) + \Psi_{0s}^{2BA}(z) + \Psi_{0s}^{3BA}(z) + \dots; \qquad z < 0 \qquad (16)
\end{aligned}$$

and

$$
\begin{aligned}
\Psi_1(z) &\overset{V\ll 1}{\approx} \Psi_1^{(0)}(z) - k_0^2 \int_0^\infty dz' G(z,z') V \Psi_1^{(0)}(z') \\
&\quad + k_0^4 \int_0^\infty dz' G(z,z') V \int_0^\infty dz'' G(z',z'') V \Psi_1^{(0)}(z'') \\
&\quad - k_0^6 \int_0^\infty dz' G(z,z') V \int_0^\infty dz'' G(z',z'') V \int_0^\infty dz''' G(z'',z''') V \Psi_1^{(0)}(z''') + \ldots \\
&\equiv \Psi_1^{(0)}(z) + \Psi_{1s}^{1BA}(z) + \Psi_{1s}^{2BA}(z) + \Psi_{1s}^{3BA}(z) + \ldots; \qquad z \geq 0 \qquad (17)
\end{aligned}
$$

where $\Psi_{0s}^{nBA}(z)$ and $\Psi_{1s}^{nBA}(z)$ ($n = 1, 2, \ldots$) are the n-th order Born approximated scattered fields in regions 0 and 1, respectively. With straightforward but tedious calculation up to the third-order Born approximation, we have

$$\Psi_{0s}^{1BA}(z) = -\frac{V}{4} e^{-ik_0 z}; \qquad z < 0, \quad (18a)$$

$$\Psi_{0s}^{2BA}(z) = \frac{V^2}{8} e^{-ik_0 z}; \qquad z < 0, \quad (18b)$$

$$\Psi_{0s}^{3BA}(z) = -\frac{5V^3}{64} e^{-ik_0 z}; \qquad z < 0, \quad (18c)$$

$$\Psi_{1s}^{1BA}(z) = \left(-\frac{V}{4} + i\frac{V}{2}k_0 z\right) e^{ik_0 z}; \qquad z \geq 0, \quad (19a)$$

$$\Psi_{1s}^{2BA}(z) = \left(\frac{V^2}{8} - i\frac{V^2}{4}k_0 z - \frac{V^2}{8}k_0^2 z^2\right) e^{ik_0 z}; \qquad z \geq 0, \quad (19b)$$

and

$$\Psi_{1s}^{3BA}(z) = \left(-\frac{5V^3}{64} + i\frac{5V^3}{32}k_0 z + \frac{3V^3}{32}k_0^2 z^2 - i\frac{V^3}{48}k_0^3 z^3\right) e^{ik_0 z}; \quad z \geq 0. \quad (19c)$$

Eqs. (18a)-(19c) are exactly the same as the corresponding terms in Eqs. (7) and (8). Under the constraint $V \ll 1$, the Born series $\sum_{n=1}^{\infty} \Psi_{0s}^{nBA}(z)$ is, but not the

Born series $\sum_{n=1}^{\infty}\Psi_{1s}^{nBA}(z)$ when $Vk_0z > 1$, uniformly valid for all z. For a fixed V and k_0, this nonuniformity problem occurred in the Born series $\sum_{n=1}^{\infty}\Psi_{1s}^{nBA}(z)$ imposes a severe limitation on the propagation distance z for the Born method to be useful.

2.3. The Rytov Method for the One-dimensional Dielectric Half-space Case

Let $\Psi_0(z)$ and $\Psi_1(z)$ be defined as

$$\Psi_0(z) \equiv \Psi_0^{(0)}(z)\, e^{i\varphi_0(z)}; \qquad z < 0 \quad (20)$$

and

$$\Psi_1(z) \equiv \Psi_1^{(0)}(z)\, e^{i\varphi_1(z)}; \qquad z \geq 0 \quad (21)$$

where $\varphi_0(z)$ and $\varphi_1(z)$ are the complex phase functions for regions 0 and 1, respectively. For the case that $k_1 = k_0\sqrt{1+V}$, exact solutions of Eqs. (3) and (4) yield exact expressions for $\varphi_0(z)$ and $\varphi_1(z)$, namely,

$$\varphi_0(z) = -\,i\,\mathrm{Ln}\left[\frac{\Psi_0(z)}{\Psi_0^{(0)}(z)}\right] = -\,i\,\mathrm{Ln}\left[1 + R(k_1)e^{-i2k_0z}\right]; \qquad z < 0 \quad (22)$$

and

$$\varphi_1(z) = -\,i\,\mathrm{Ln}\left[\frac{\Psi_1(z)}{\Psi_1^{(0)}(z)}\right] = -\,i\,\mathrm{Ln}[T(k_1)] + (k_1 - k_0)z; \qquad z \geq 0 \quad (23)$$

where Ln[X] is the natural logarithm of X. If $V \ll 1$, we can expand $\varphi_0(z)$ and $\varphi_1(z)$ into Taylor series in terms of V, i.e.,

$$\varphi_0(z) \overset{V\ll 1}{\approx} i\left(\frac{V}{4} - \frac{V^2}{8} + \frac{5V^3}{64}\right)e^{-i2k_0z} + i\left(\frac{V^2}{32} - \frac{V^3}{32}\right)e^{-i4k_0z} + i\,\frac{V^3}{192}e^{-i6k_0z} + \ldots; \qquad z < 0 \quad (24)$$

and

$$\varphi_1(z) \overset{V\ll 1}{\approx} i\left(\frac{V}{4} - \frac{3V^2}{32} + \frac{5V^3}{96}\right) + \left(\frac{V}{2} - \frac{V^2}{8} + \frac{V^3}{16}\right) k_0 z + \ldots; \qquad z \geq 0. \qquad (25)$$

Under the definitions of Eqs. (20) and (21), we rearrange the wave equations in Eqs. (9) and (10) and obtain [35,64]

$$\frac{d^2}{dz^2}\left[\Psi_0^{(0)}(z)\varphi_0(z)\right] + k_0^2\,\Psi_0^{(0)}(z)\varphi_0(z) = -\,i\left[\frac{d\varphi_0(z)}{dz}\right]^2 \Psi_0^{(0)}(z); \qquad z < 0 \qquad (26)$$

and

$$\frac{d^2}{dz^2}\left[\Psi_1^{(0)}(z)\varphi_1(z)\right] + k_0^2\,\Psi_1^{(0)}(z)\varphi_1(z) = i\,k_0^2\,V\Psi_1^{(0)}(z) - i\left[\frac{d\varphi_1(z)}{dz}\right]^2 \Psi_1^{(0)}(z);$$

$$z \geq 0. \qquad (27)$$

Therefore, with the application of the 1-D unbounded scalar Green's function $G(z, z')$, we can express $\varphi_0(z)$ and $\varphi_1(z)$ as

$$\varphi_0(z) = -\frac{i}{\Psi_0^{(0)}(z)} \int_{-\infty}^{0} dz'\,G(z,z')\left[\frac{d\varphi_0(z')}{dz'}\right]^2 \Psi_0^{(0)}(z')$$

$$+\frac{i}{\Psi_0^{(0)}(z)} \int_{0}^{\infty} dz'\,G(z,z')\left\{k_0^2 V - \left[\frac{d\varphi_1(z')}{dz'}\right]^2\right\}\Psi_1^{(0)}(z'); \qquad z < 0 \qquad (28)$$

and

$$\varphi_1(z) = -\frac{i}{\Psi_1^{(0)}(z)} \int_{-\infty}^{0} dz'\,G(z,z')\left[\frac{d\varphi_0(z')}{dz'}\right]^2 \Psi_0^{(0)}(z')$$

$$+\frac{i}{\Psi_1^{(0)}(z)} \int_{0}^{\infty} dz'\,G(z,z')\left\{k_0^2 V - \left[\frac{d\varphi_1(z')}{dz'}\right]^2\right\}\Psi_1^{(0)}(z'); \qquad z \geq 0. \qquad (29)$$

It is easy to show that the exact solutions of $\varphi_0(z)$ and $\varphi_1(z)$ in Eqs. (22) and (23) can also be derived from Eqs. (28) and (29) by using Eq. (15) and the exact phase changes;

$$\frac{d\varphi_0(z)}{dz} = -\frac{2k_0R(k_1)\,e^{-i2k_0z}}{1 + R(k_1)\,e^{-i2k_0z}}, \qquad z < 0 \qquad (30)$$

and

$$\frac{d\varphi_1(z)}{dz} = k_1 - k_0; \qquad z \geq 0. \qquad (31)$$

Eqs. (24) and (25) can also be derived from the Rytov-approximated solutions of Eqs. (28) and (29), i.e.,

$$\varphi_0(z) \approx \frac{ik_0^2}{\Psi_0^{(0)}(z)} \int_0^\infty dz' G(z,z') V \Psi_1^{(0)}(z')$$

$$- \frac{i}{\Psi_0^{(0)}(z)} \int_{-\infty}^0 dz' G(z,z') \left[\frac{d\varphi_0^{1RA}(z')}{dz'} + \frac{d\varphi_0^{2RA}(z')}{dz'} + \frac{d\varphi_0^{3RA}(z')}{dz'} + \ldots\right]^2 \Psi_0^{(0)}(z')$$

$$- \frac{i}{\Psi_0^{(0)}(z)} \int_0^\infty dz' G(z,z') \left[\frac{d\varphi_1^{1RA}(z')}{dz'} + \frac{d\varphi_1^{2RA}(z')}{dz'} + \frac{d\varphi_1^{3RA}(z')}{dz'} + \ldots\right]^2 \Psi_1^{(0)}(z')$$

$$= \varphi_0^{1RA}(z) + \varphi_0^{2RA}(z) + \varphi_0^{3RA}(z) + \ldots; \qquad z < 0 \qquad (32)$$

and

$$\varphi_1(z) \approx \frac{ik_0^2}{\Psi_1^{(0)}(z)} \int_0^\infty dz' G(z,z') V \Psi_1^{(0)}(z')$$

$$
-\frac{i}{\Psi_1^{(0)}(z)}\int_{-\infty}^{0} dz' G(z,z')\left[\frac{d\varphi_0^{1RA}(z')}{dz'}+\frac{d\varphi_0^{2RA}(z')}{dz'}+\frac{d\varphi_0^{3RA}(z')}{dz'}+\ldots\right]^2 \Psi_0^{(0)}(z')
$$

$$
-\frac{i}{\Psi_1^{(0)}(z)}\int_{0}^{\infty} dz' G(z,z')\left[\frac{d\varphi_1^{1RA}(z')}{dz'}+\frac{d\varphi_1^{2RA}(z')}{dz'}+\frac{d\varphi_1^{3RA}(z')}{dz'}+\ldots\right]^2 \Psi_1^{(0)}(z')
$$

$$
=\varphi_1^{1RA}(z)+\varphi_1^{2RA}(z)+\varphi_1^{3RA}(z)+\ldots; \qquad z \geq 0 \qquad (33)
$$

where $\varphi_0^{nRA}(z)$ and $\varphi_1^{nRA}(z)$ are the n-th order Rytov approximated complex phase functions in regions 0 and 1, respectively. After lengthy but simple calculations, we obtain

$$
\varphi_0^{1RA}(z)=\frac{ik_0^2}{\Psi_0^{(0)}(z)}\int_0^{\infty} dz' G(z,z') V \Psi_1^{(0)}(z')
$$

$$
= i\,\frac{V}{4}\,e^{-i2k_0z}; \qquad z < 0, \qquad (34a)
$$

$$
\varphi_1^{1RA}(z)=\frac{ik_0^2}{\Psi_1^{(0)}(z)}\int_0^{\infty} dz' G(z,z') V \Psi_1^{(0)}(z')
$$

$$
= i\,\frac{V}{4}+\frac{V}{2}\,k_0 z; \qquad z \geq 0, \qquad (35a)
$$

$$
\varphi_0^{2RA}(z) = -\frac{i}{\Psi_0^{(0)}(z)}\int_{-\infty}^{0} dz' G(z,z')\left[\frac{d\varphi_0^{1RA}(z')}{dz'}\right]^2 \Psi_0^{(0)}(z')
$$

$$
-\frac{i}{\Psi_0^{(0)}(z)}\int_{0}^{\infty} dz' G(z,z')\left[\frac{d\varphi_1^{1RA}(z')}{dz'}\right]^2 \Psi_1^{(0)}(z')
$$

$$= -i\frac{V^2}{8}e^{-i2k_0z} + i\frac{V^2}{32}e^{-i4k_0z}; \qquad z<0. \quad (34b)$$

$$\varphi_1^{2RA}(z) = -\frac{i}{\Psi_1^{(0)}(z)}\int_{-\infty}^{0}dz'G(z,z')\left[\frac{d\varphi_0^{1RA}(z')}{dz'}\right]^2\Psi_0^{(0)}(z')$$

$$-\frac{i}{\Psi_1^{(0)}(z)}\int_{0}^{\infty}dz'G(z,z')\left[\frac{d\varphi_1^{1RA}(z')}{dz'}\right]^2\Psi_1^{(0)}(z')$$

$$= -i\frac{3V^2}{32} - \frac{V^2}{8}k_0z; \qquad z \geq 0, \quad (35b)$$

$$\varphi_0^{3RA}(z) = -\frac{i2}{\Psi_0^{(0)}(z)}\int_{-\infty}^{0}dz'G(z,z')\left[\frac{d\varphi_0^{1RA}(z')}{dz'}\right]\left[\frac{d\varphi_0^{2RA}(z')}{dz'}\right]\Psi_0^{(0)}(z')$$

$$-\frac{i2}{\Psi_0^{(0)}(z)}\int_{0}^{\infty}dz'G(z,z')\left[\frac{d\varphi_1^{1RA}(z')}{dz'}\right]\left[\frac{d\varphi_1^{2RA}(z')}{dz'}\right]\Psi_1^{(0)}(z')$$

$$= i\frac{5V^3}{64}e^{-i2k_0z} - i\frac{V^3}{32}e^{-i4k_0z} + i\frac{V^3}{192}e^{-i6k_0z}; \qquad z<0, \quad (34c)$$

and

$$\varphi_1^{3RA}(z) = -\frac{i2}{\Psi_1^{(0)}(z)}\int_{-\infty}^{0}dz'G(z,z')\left[\frac{d\varphi_0^{1RA}(z')}{dz'}\right]\left[\frac{d\varphi_0^{2RA}(z')}{dz'}\right]\Psi_0^{(0)}(z')$$

$$-\frac{i2}{\Psi_1^{(0)}(z)}\int_{0}^{\infty}dz'G(z,z')\left[\frac{d\varphi_1^{1RA}(z')}{dz'}\right]\left[\frac{d\varphi_1^{2RA}(z')}{dz'}\right]\Psi_1^{(0)}(z')$$

$$= i\frac{5V^3}{96} + \frac{V^3}{16}k_0z; \qquad z \geq 0. \quad (35c)$$

The factor of two in Eqs. (34c) and (35c) is essential for the derivation. With increasingly cumbersome formulations, one can follow the same iteration scheme to derive higher-order terms of the Rytov series for $\varphi_0(z)$ and $\varphi_1(z)$. Since the perturbation expansions for $\varphi_m(z)$ in Eqs. (32) and (33) are based on the criteria that $|\varphi_m^{nRA}(z)| \ll |\varphi_m^{(n-1)RA}(z)|$ (m = 0, 1 and n = 1, 2, ...) which, in turn, leads to the constraint $V \ll 1$, it imposes no restriction on the propagation distance z for the Rytov method.

From Eqs. (34a)-(35c), we have shown that the conventional way of omitting $[d\varphi_0(z)/dz]^2$ and $[d\varphi_1(z)/dz]^2$ in Eqs. (24) and (25) under the Rytov approximation [1,2,6] is equivalent to neglect the multiple scattering processes [11] which correspond to higher-order terms (n > 1) in Eqs. (32) and (33). Also, we have shown that the omission of $[d\varphi_0(z)/dz]^2$ and $[d\varphi_1(z)/dz]^2$ in the Rytov approximation has nothing to do with the magnitudes of the complex phase functions as long as the contributions from $[d\varphi_0(z)/dz]^2$ and $[d\varphi_1(z)/dz]^2$ are negligible compared with k_0^2V [Ref. 17, Appendix A]. Therefore, the argument [7,29,32] that the Rytov approximation is not valid when $|\varphi_0(z)| \sim 1$ and $|\varphi_1(z)| \sim 1$ is incorrect.

2.4. Relationship between the Born and the Rytov Methods

We have recovered, from Eqs. (18a)-(19c) and (34a)-(35c), the well-known relations between the Born-approximated scattered fields and the Rytov-approximated complex phase functions [25-27], namely,

$$\Psi_{ms}^{1BA}(z) = i\Psi_m^{(0)}(z)\varphi_m^{1RA}(z), \tag{36a}$$

$$\Psi_{ms}^{2BA}(z) = \Psi_m^{(0)}(z)\left\{\frac{\left[i\varphi_m^{1RA}(z)\right]^2}{2!} + i\varphi_m^{2RA}(z)\right\}, \tag{36b}$$

$$\Psi_{ms}^{3BA}(z) = \Psi_m^{(0)}(z)\left\{\frac{\left[i\varphi_m^{1RA}(z)\right]^3}{3!} - \varphi_m^{1RA}(z)\varphi_m^{2RA}(z) + i\varphi_m^{3RA}(z)\right\}, \tag{36c}$$

and so on for higher-order terms in region m (0 and 1). In order to compare domains of validity for both Born and Rytov methods, we examine the relationship between the Born and the Rytov solutions shown below in Eqs. (37a)-(38c) where, up to the third-order power of V, the scattered fields derived by the Rytov method can be reduced to that of derived by the Born method, i.e.,

$$\Psi_{0s}^{1RA}(z) = \Psi_0^{(0)}(z)\left[e^{i\varphi_0^{1RA}(z)} - 1\right]$$

$$\overset{V \ll 1}{\approx} -\frac{V}{4}e^{-ik_0z} + O\left(V^2e^{-i3k_0z}\right)$$

$$= \Psi_{0s}^{1BA}(z) + O\left(V^2e^{-i3k_0z}\right), \qquad z < 0, \tag{37a}$$

$$\Psi_{0s}^{(1+2)RA}(z) = \Psi_0^{(0)}(z)\left\{e^{i\left[\varphi_0^{1RA}(z) + \varphi_0^{2RA}(z)\right]} - 1\right\}$$

$$\overset{V \ll 1}{\approx} \left(-\frac{V}{4} + \frac{V^2}{8}\right)e^{-ik_0z} + O\left(V^3e^{-i3k_0z}\right)$$

$$= \Psi_{0s}^{1BA}(z) + \Psi_{0s}^{2BA}(z) + O\left(V^3e^{-i3k_0z}\right), \qquad z < 0, \tag{37b}$$

$$\Psi_{0s}^{(1+2+3)RA}(z) = \Psi_0^{(0)}(z)\left\{e^{i\left[\varphi_0^{1RA}(z) + \varphi_0^{2RA}(z) + \varphi_0^{3RA}(z)\right]} - 1\right\}$$

$$\overset{V \ll 1}{\approx} \left(-\frac{V}{4} + \frac{V^2}{8} - \frac{5V^3}{64}\right)e^{-ik_0z} + O\left(V^4e^{-i3k_0z}\right)$$

$$= \Psi_{0s}^{1BA}(z) + \Psi_{0s}^{2BA}(z) + \Psi_{0s}^{3BA}(z) + O\left(V^4 e^{-i3k_0 z}\right); \qquad z < 0, \quad (37c)$$

$$\Psi_{1s}^{1RA}(z) = \Psi_1^{(0)}(z)\left[e^{i\varphi_1^{1RA}(z)} - 1\right]$$

$$\stackrel{V\ll 1}{\approx} \left(-\frac{V}{4} + i\frac{V}{2}k_0 z\right)e^{ik_0 z} + O(V^2) + O\left(V^2 k_0^2 z^2\right)$$

$$= \Psi_{1s}^{1BA}(z) + O(V^2) + O\left(V^2 k_0^2 z^2\right); \qquad z \geq 0, \quad (38a)$$

$$\Psi_{1s}^{(1+2)RA}(z) = \Psi_1^{(0)}(z)\left\{e^{i\left[\varphi_1^{1RA}(z) + \varphi_1^{2RA}(z)\right]} - 1\right\}$$

$$\stackrel{V\ll 1}{\approx} \left(-\frac{V}{4} + i\frac{V}{2}k_0 z + \frac{V^2}{8} - i\frac{V^2}{4}k_0 z - \frac{V^2}{8}k_0^2 z^2\right)e^{ik_0 z} + O(V^3) + O\left(V^3 k_0^2 z^2\right)$$

$$= \Psi_{1s}^{1BA}(z) + \Psi_{1s}^{2BA}(z) + O(V^3) + O\left(V^3 k_0^2 z^2\right); \qquad z \geq 0, \quad (38b)$$

and

$$\Psi_{1s}^{(1+2+3)RA}(z) = \Psi_1^{(0)}(z)\left\{e^{i\left[\varphi_1^{1RA}(z) + \varphi_1^{2RA}(z) + \varphi_1^{3RA}(z)\right]} - 1\right\}$$

$$\stackrel{V\ll 1}{\approx} \left(-\frac{V}{4} + i\frac{V}{2}k_0 z + \frac{V^2}{8} - i\frac{V^2}{4}k_0 z - \frac{V^2}{8}k_0^2 z^2 - \frac{5V^3}{64} + i\frac{5V^3}{32}k_0 z + \frac{3V^3}{32}k_0^2 z^2\right.$$

$$\left. - i\frac{V^3}{48}k_0^3 z^3\right) e^{ik_0 z} + O(V^4) + O\left(V^4 k_0^2 z^2\right)$$

$$= \Psi_{1s}^{1BA}(z) + \Psi_{1s}^{2BA}(z) + \Psi_{1s}^{3BA}(z) + O(V^4) + O\left(V^4 k_0^2 z^2\right); \qquad z \geq 0. \quad (38c)$$

In the Rytov method, the scattered fields corresponding to the Rytov series $\sum_{j=1}^{n}\varphi_m^{jRA}(z)$ is denoted by $\Psi_{ms}^{(1+2+...+n)RA}(z) = \Psi_m^{(0)}(z)\left\{\exp[i\sum_{j=1}^{n}\varphi_m^{jRA}(z)] - 1\right\}$ (m = 0, 1). From Eqs. (37a)-(38c), it is shown that the multiple scattering up to the n-th order is included in the Taylor expansions of $\Psi_{ms}^{(1+2+...+n)RA}(z)$, which can be

represented by the sum of $\sum_{j=1}^{n}\Psi_{ms}^{jBA}(z)$ plus an infinite number of redundant higher-order terms. As implied by Eqs. (37a) and (38a), it is incorrect to address the Taylor expansion of $\exp[\rho_1(\mathbf{r})]$ [Eq. (12) of Ref. 13] as "a multiple-scattering interpretation of the Rytov solution" because, for example, the quantity $\rho_1^2(\mathbf{r}) = [2k_0^2\int_v dv'G(\mathbf{r},\mathbf{r}')N(\mathbf{r}')A_0(\mathbf{r}')/A_0(\mathbf{r})]^2$, which is the square of $\rho_1(\mathbf{r})$ representing the single scattering process, does not correspond to the double scattering process whereas the quantity $4k_0^4\int_v dv'G(\mathbf{r},\mathbf{r}')N(\mathbf{r}')\int_v dv''G(\mathbf{r}',\mathbf{r}'')N(\mathbf{r}'')A_0(\mathbf{r}'')/A_0(\mathbf{r})$ does. Also, from Eqs. (37a)-(37c), we have shown that for finite n, $\Psi_{0s}^{(1+2+...+n)RA}(z)$ is inadequate to approximate the scattered field $\Psi_{0s}(z)$ (or the reflected field in free-space) because it does not provide the correct propagation factor e^{-ik_0z} whereas the Born series $\sum_{j=1}^{n}\Psi_{0s}^{jBA}(z)$ does [as stated in Ref. 41 for n = 1]. Hence, although both methods have the same domain of validity (defined by the constraint $V \ll 1$) when each is applied to estimate $\Psi_{0s}(z)$, the Born method is superior to the Rytov method. On the other hand, to estimate the scattered field $\Psi_{1s}(z)$ in the dielectric half-space, the Rytov method is superior to the Born method because the Born series $\sum_{n=1}^{\infty}\Psi_{1s}^{nBA}(z)$ suffers the nonuniformity in convergence if $Vk_0z > 1$, which is not the case with the Rytov method.

Note that the redundant terms in the Taylor expansions of $\Psi_{ms}^{(1+2+...+n)RA}(z)$ (m = 0, 1) cancel out one another exactly if $n \to \infty$. Thus, we have

$$\lim_{n\to\infty}\Psi_{ms}^{(1+2+...+n)RA}(z) = \Psi_m^{(0)}(z)\left\{\exp\left[i\sum_{j=1}^{\infty}\varphi_m^{jRA}(z)\right] - 1\right\} \overset{V\ll 1}{\approx} \sum_{j=1}^{\infty}\Psi_{ms}^{jBA}(z). \qquad (39)$$

3. One-dimensional Homogeneous Dielectric Slab

In this case, the same plane wave as that described in section 2 is incident from free-space (region 0, $z < 0$) into the homogeneous dielectric slab (region 1, $0 \le z \le d$) with permeability μ_0 and permittivity $\varepsilon_0(1 + V)$ [see Figure 2]. The boundary conditions at $z = 0$ and $z = d$ are

$$\Psi_0(0) = \Psi_1(0), \tag{40a}$$

$$\Psi_1(d) = \Psi_2(d), \tag{40b}$$

$$\left.\frac{d\Psi_0(z)}{dz}\right|_{z=0} = \left.\frac{d\Psi_1(z)}{dz}\right|_{z=0}, \tag{41a}$$

and

$$\left.\frac{d\Psi_1(z)}{dz}\right|_{z=d} = \left.\frac{d\Psi_2(z)}{dz}\right|_{z=d} \tag{41b}$$

where $\Psi_2(z)$ is the total field in region 2 (free-space for $z > d$). By matching the boundary conditions, the total fields in three regions are given as

$$\Psi_0(z) = e^{ik_0z} + \frac{R(k_1)\left(1 - e^{i2k_1d}\right)}{D(k_1)}e^{-ik_0z}; \qquad z < 0, \tag{42}$$

$$\Psi_1(z) = \frac{T(k_1)}{D(k_1)}e^{ik_1z} - \frac{R(k_1)T(k_1)e^{i2k_1d}}{D(k_1)}e^{-ik_1z}; \qquad 0 \le z \le d, \tag{43}$$

and

$$\Psi_2(z) = \frac{T(k_1)[1 - R(k_1)]e^{i(k_1 - k_0)d}}{D(k_1)}e^{ik_0z}; \qquad z > d, \tag{44}$$

where $D(k_1) \equiv 1 - [R(k_1)]^2 e^{i2k_1d}$. To reduce the total fields in Eqs. (42) and (43) for the dielectric-slab case to that in Eqs. (3) and (4) for the dielectric half-space case in which $\Psi_2(z)$ does not exist, one can invoke the principle of limiting absorption [41] to assume that k_1 contains a small and positive imaginary part

such that $\lim_{d\to\infty} e^{ik_1 d} = 0$. If $V \ll 1$ and $Vk_0 d \ll 1$, Eqs. (42)-(44) can be expanded into the Taylor series:

$$\Psi_0(z) \overset{\substack{V\ll 1 \\ Vk_0d\ll 1}}{\approx} e^{ik_0z} + \left\{\left[-\frac{V}{4} + \frac{V^2}{8} - \frac{5V^3}{64} - \frac{V^3}{64}e^{i2k_0d}\right]\left(1 - e^{i2k_0d}\right)\right.$$

$$\left. + i\left(\frac{V^2}{4} - \frac{3V^3}{16}\right)k_0d\, e^{i2k_0d} - \frac{V^3}{8}k_0^2\, d^2\, e^{i2k_0d} + \ldots\right\} e^{-ik_0z}; \qquad z < 0, \qquad (45)$$

$$\Psi_1(z) \overset{\substack{V\ll 1 \\ Vk_0d\ll 1}}{\approx} \left[1 - \frac{V}{4} + \frac{V^2}{8} - \frac{5V^3}{64} + \left(\frac{V^2}{16} - \frac{5V^3}{64}\right)e^{i2k_0d} + i\,\frac{V^3}{16}\,k_0\left(d + \frac{z}{2}\right) e^{i2k_0d}\right.$$

$$\left. + i\left(\frac{V}{2} - \frac{V^2}{4} + \frac{5V^3}{32}\right)k_0z + \left(-\frac{V^2}{8} + \frac{3V^3}{32}\right)k_0^2\, z^2 - i\,\frac{V^3}{48}k_0^3\, z^3\right] e^{ik_0z}$$

$$+ \left[\frac{V}{4} - \frac{3V^2}{16} + \frac{9V^3}{64} + \frac{V^3}{64}e^{i2k_0d} + i\left(\frac{V^2}{4} - \frac{V^3}{4}\right)k_0\left(d - \frac{z}{2}\right)\right.$$

$$\left. - \frac{V^3}{8}k_0^2\left(d - \frac{z}{2}\right)^2\right] e^{i2k_0d}\, e^{-ik_0z} + \ldots; \qquad 0 \le z \le d, \qquad (46)$$

and

$$\Psi_2(z) \overset{\substack{V\ll 1 \\ Vk_0d\ll 1}}{\approx} \left[1 + \left(-\frac{V^2}{16} + \frac{V^3}{16}\right)\left(1 - e^{i2k_0d}\right) + i\left(\frac{V}{2} - \frac{V^2}{8} + \frac{V^3}{32}\right)k_0d\right.$$

$$\left. + i\,\frac{3V^3}{32}k_0d\, e^{i2k_0d} + \left(-\frac{V^2}{8} + \frac{V^3}{16}\right)k_0^2\, d^2 - i\,\frac{V^3}{48}k_0^3\, d^3 + \ldots\right] e^{ik_0z}; \qquad z > d. \qquad (47)$$

One can also derive Eqs. (42)-(44) by using the inhomogeneous Fredholm equations of the first kind for the total fields in region 0, 1, and 2, namely,

$$\Psi_m(z) = \Psi_m^{(0)}(z) - k_0^2 \int_0^d dz'\, G(z, z')\, V\Psi_1(z'); \qquad (m = 0, 1, 2) \qquad (48)$$

where $G(z, z')$ and $\Psi_1(z)$ are given in Eqs. (15) and (43), respectively, and $\Psi_0^{(0)}(z) = \Psi_1^{(0)}(z) = \Psi_2^{(0)}(z) = e^{ik_0 z}$. With tedious but straightforward calculations, it can be verified that successive terms in the Born series, derived iteratively by replacing $\Psi_1(z)$ with $\Psi_1^{(0)}(z)$ in Eq. (48), are exactly equal to the corresponding terms in the Taylor-series expansions for Eqs. (42)-(44) in terms of V.

Since only the constraint $V \ll 1$ is used in the Born method for estimating $\Psi_m(z)$ (m = 0, 1, 2), all of three Born series $\sum_{n=1}^{\infty} \Psi_{ms}^{nBA}(z)$ diverge if $Vk_0d > 1$. The Born series $\sum_{n=1}^{\infty} \Psi_{1s}^{nBA}(z)$ is subject to two special situations:

(1) if $z \approx 0$, $\sum_{n=1}^{\infty} \Psi_{1s}^{nBA}(z)$ behaves like $\sum_{n=1}^{\infty} \Psi_{0s}^{nBA}(z)$ and

(2) if $z \approx d$, $\sum_{n=1}^{\infty} \Psi_{1s}^{nBA}(z)$ behaves like $\sum_{n=1}^{\infty} \Psi_{2s}^{nBA}(z)$.

Therefore, to obtain good estimates for $\Psi_m(z)$ by using the Born approximation requires that $V \ll 1$ and $Vk_0d \ll 1$. These two constraints impose a severe limitation on the domain of validity for the Born method.

Alternatively, $\Psi_0(z)$, $\Psi_1(z)$, and $\Psi_2(z)$ can be defined as:

$$\Psi_m(z) \equiv \Psi_m^{(0)}(z)\, e^{i\varphi_m(z)} \tag{49}$$

where $\Psi_m^{(0)}(z)$ and $\varphi_m(z)$ (m = 0, 1, 2) are the unperturbed fields and the complex phase functions for regions 0 ($z < 0$), 1 ($0 \le z \le d$), and 2 ($z > d$), respectively. By the exact solutions of Eqs (42)-(44), we have

$$\varphi_0(z) = -\, i\, \mathrm{Ln}\left[1 + \frac{R(k_1)\left(1 - e^{i2k_1 d}\right)}{D(k_1)}\, e^{-i2k_0 z} \right], \qquad z < 0, \tag{50}$$

$$\varphi_1(z) = -\, i\, \mathrm{Ln}\left[\frac{T(k_1)}{D(k_1)} - \frac{R(k_1)T(k_1)e^{i2k_1d}}{D(k_1)}\, e^{-i2k_1z}\right] + (k_1 - k_0)z; \qquad 0 \le z \le d. \quad (51)$$

and

$$\varphi_2(z) = -\, i\, \mathrm{Ln}\left\{\frac{T(k_1)[1 - R(k_1)]}{D(k_1)}\right\} + (k_1 - k_0)d; \qquad z > d. \quad (52)$$

When $V \ll 1$ and $Vk_0d \ll 1$, Eqs. (50)-(52) can be expanded into the Taylor series in terms of V, namely,

$$\varphi_0(z) \overset{\substack{V\ll 1\\ Vk_0d\ll 1}}{\approx} \Big[i\left(\frac{V}{4} - \frac{V^2}{8} + \frac{5V^3}{64} + \frac{V^3}{64}e^{i2k_0d}\right)\left(1 - e^{i2k_0d}\right)$$

$$+ \left(\frac{V^2}{4} - \frac{3V^3}{16}\right)k_0d\, e^{i2k_0d} + i\,\frac{V^3}{8}k_0^2\, d^2\, e^{i2k_0d}\Big]\, e^{-i2k_0z}$$

$$+ \left[i\left(\frac{V^2}{32} - \frac{V^3}{32}\right)\left(1 - e^{i2k_0d}\right) + \frac{V^3}{16}k_0d\, e^{i2k_0d}\right]\left(1 - e^{i2k_0d}\right)e^{-i4k_0z}$$

$$+ i\,\frac{V^3}{192}\left(1 - e^{i2k_0d}\right)^3 e^{-i6k_0z} + \ldots; \qquad z < 0, \quad (53)$$

$$\varphi_1(z) \overset{\substack{V\ll 1\\ Vk_0d\ll 1}}{\approx} i\left(\frac{V}{4} - \frac{3V^2}{32} + \frac{5V^3}{96}\right) + \left(-\, i\,\frac{V^2}{16} + i\,\frac{V^3}{16} + \frac{V^3}{16}k_0d\right)e^{i2k_0d}$$

$$+ \left(\frac{V}{2} - \frac{V^2}{8} + \frac{V^3}{16}\right)k_0z + \Big[i\left(-\,\frac{V}{4} + \frac{V^2}{8} - \frac{5V^3}{64}\right) + \left(\frac{V^2}{4} + \frac{V^3}{16}\right)k_0(d - z)$$

$$+ i\,\frac{V^3}{8}k_0^2(d - z)^2\Big]\, e^{i2k_0d}\, e^{-i2k_0z} + \left[i\left(\frac{V^2}{32} - \frac{V^3}{32}\right) - \frac{V^3}{16}k_0(d - z)\right]e^{i4k_0d}\, e^{-i4k_0z}$$

$$-\, i\,\frac{V^3}{192}e^{i6k_0d}\, e^{-i6k_0z} + O(V^4) + O\left(V^4k_0^2d^2\right) + O\left[V^4k_0^3(d - z)^3\right] + \ldots;$$

$$0 \le z \le d, \quad (54)$$

and

$$\varphi_2(z) \overset{\substack{V\ll 1\\ Vk_0d\ll 1}}{\approx} \left(\frac{V}{2} - \frac{V^2}{8} + \frac{V^3}{16} - \frac{5V^4}{128}\right)k_0d + i\left(\frac{V^2}{16} - \frac{V^3}{16} + \frac{29V^4}{512}\right)$$

$$+ \left[i \left(- \frac{V^2}{16} + \frac{V^3}{16} - \frac{7V^4}{128} \right) + \left(\frac{V^3}{16} + \frac{5V^4}{64} \right) k_0 d + i \frac{V^4}{32} k_0^2 d^2 \right] e^{i2k_0 d}$$

$$- i \frac{V^4}{512} e^{i4k_0 d} + \ldots; \qquad z > d. \quad (55)$$

As usual, Eqs. (50)-(52) can also be derived from the following integral representations, i.e.,

$$\varphi_m(z) = - \frac{i}{\Psi_m^{(0)}(z)} \int_{-\infty}^{0} dz' G(z, z') \left[\frac{d\varphi_0(z')}{dz'} \right]^2 \Psi_0^{(0)}(z')$$

$$+ \frac{i}{\Psi_m^{(0)}(z)} \int_{0}^{d} dz' G(z, z') \left\{ k_0^2 V - \left[\frac{d\varphi_1(z')}{dz'} \right]^2 \right\} \Psi_1^{(0)}(z')$$

$$- \frac{i}{\Psi_m^{(0)}(z)} \int_{d}^{\infty} dz' G(z, z') \left[\frac{d\varphi_2(z')}{dz'} \right]^2 \Psi_2^{(0)}(z'); \qquad (m = 0, 1, 2) \quad (56)$$

By the same token as that for Eqs. (34a)-(35c), Eqs. (53)-(55) can be derived from Eq. (56) by iteration. Hence, as in Eqs. (36a)-(36c), the same well-known relations between the Born and the Rytov series can also be established in all three regions. From Eqs. (53)-(55), it is shown that the Rytov series $\sum_{n=1}^{\infty} \varphi_1^{nRA}(z)$ is also subject to two special situations:

(1) if $z \approx 0$, $\sum_{n=1}^{\infty} \varphi_1^{nRA}(z)$ behaves like $\sum_{n=1}^{\infty} \varphi_0^{nRA}(z)$ and

(2) if $z \approx d$, $\sum_{n=1}^{\infty} \varphi_1^{nRA}(z)$ behaves like $\sum_{n=1}^{\infty} \varphi_2^{nRA}(z)$.

If the Rytov approximation is able to provide fair estimates for $\varphi_m(z)$, both constraints that $V \ll 1$ and $Vk_0 d \ll 1$ should be invoked. Hence, the domain of

validity for the Rytov method is the same as that for the Born method in this dielectric-slab case.

It is worth pointing out that if we let the thickness d of the dielectric slab approach to infinity but keep V and k_0 fixed, we are unable to establish the same statement as that in the dielectric half-space case on issues of applicability and domains of validity for the Born and the Rytov methods. This is attributed to the fact that we have invoked the principle of limiting absorption to neglect the highly oscillating terms of the form $\lim_{z\to\infty} e^{ik_0z}$ which should exist after the integrations are carried out for the dielectric-slab case, even when d is very large (but finite).

4. Conclusion and Remarks

We have shown that, to approximate the scattered field (or the reflected field) in free-space of the dielectric half-space case, the Born method is more suitable than the Rytov method although both have the same domain of validity. But, to compute the scattered field in the dielectric half-space, the Rytov method is superior to the Born method because the Born series $\sum_{n=1}^{\infty}\Psi_{1s}^{nBA}(z)$ suffers the nonuniformity in convergence when $Vk_0z > 1$. In the dielectric-slab case, both Born and Rytov methods have the same (and very narrow) domain of validity. However, when $Vk_0d > 1$, these two methods are inapplicable because the Born and the Rytov series diverge. Despite of the fact that the principle of limiting absorption provides a self-consistent way of deriving the exact total fields for both 1-D deterministic cases, it is unable to eliminate the secular terms occurring in the Rytov series for the dielectric-slab case when d becomes very large such that $Vk_0d > 1$. In summary, the domains of validity for the Born and

the Rytov approximations are described in tables 1 and 2 for the dielectric half-space and the dielectric-slab cases, respectively.

The above statements concerning the Born and the Rytov methods are also valid for the situation when the dielectric half-space or the dielectric slab has a permittivity of the form $\varepsilon_1(1 + V)$ where $\varepsilon_1 > \varepsilon_0$. In this case, to derive the Born and the Rytov series from inhomogeneous Fredholm equations of the first kind requires a set of appropriate Green's functions which satisfy the boundary conditions at the planar interfaces. We have shown that with assistance of these sets of Green's functions for 1-D deterministic cases, we are able to develop the correct integral representations, which are beneficial to both direct and inverse scattering theory, for the exact total fields in different regions [70]. Note that, in conjunction with the application of the statistical theory, the above theoretical treatments for 1-D deterministic cases can be extended to that for 1-D stochastic cases. In addition, understanding the physical interpretations inferred from analytical expressions for these 1-D cases will shed some light on the scattering theory for 3-D cases where no closed-form expressions exist.

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Figure Captions

Figure 1. Scattering geometry for 1-D dielectric half-space case.

Figure 2. Scattering geometry for 1-D dielectric-slab case.

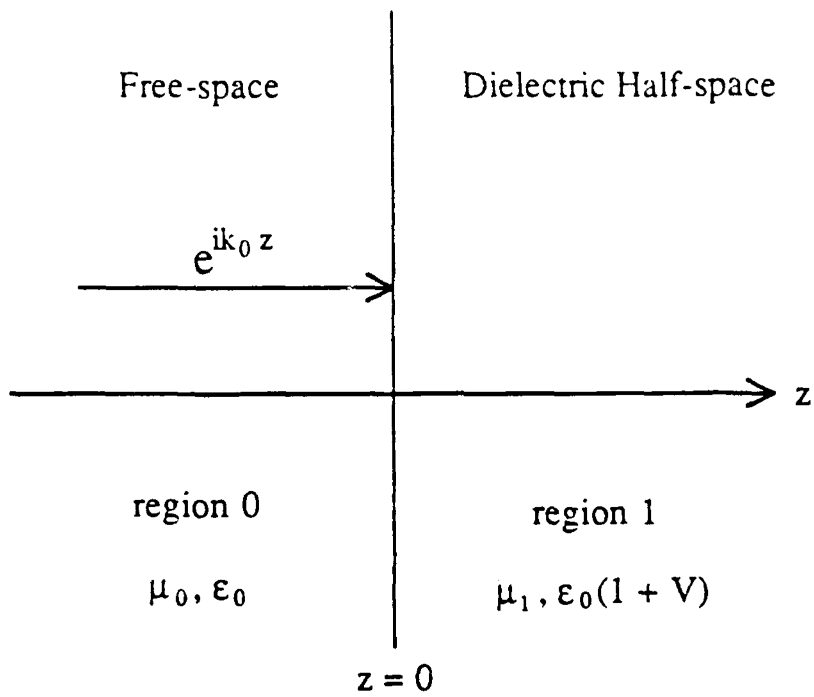
Free-space
Dielectric Half-space
$e^{ik_0 z}$
z
region 0
region 1
μ_0, ε_0
$\mu_1, \varepsilon_0(1 + V)$
z = 0

Figure 1.

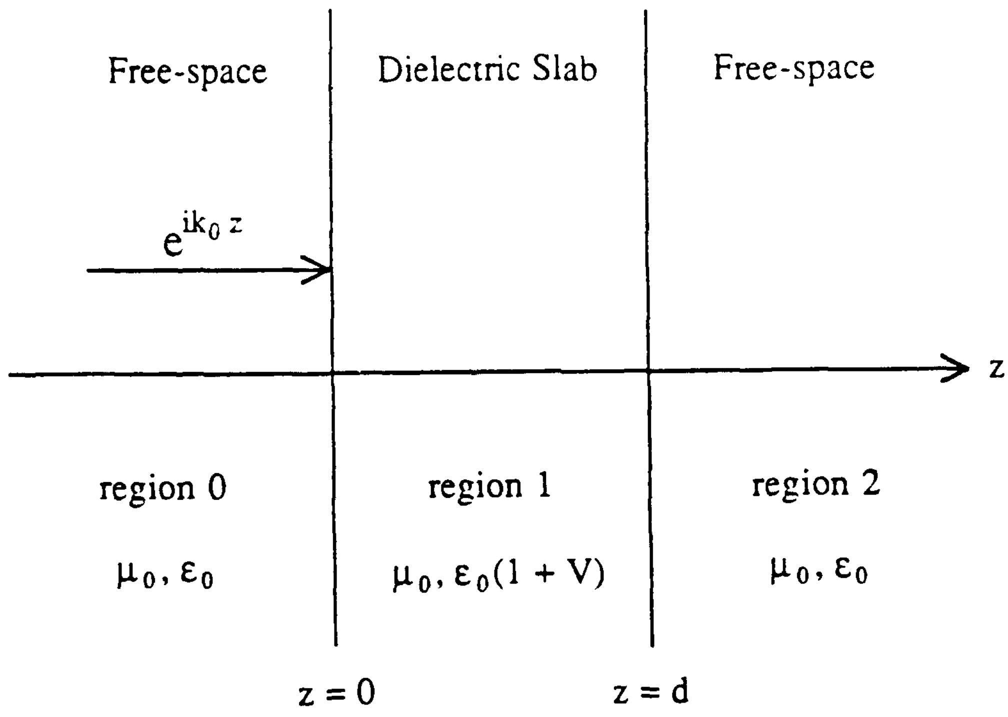
Free-space
Dielectric Slab
Free-space
$e^{ik_0 z}$
z
region 0
region 1
region 2
μ_0, ε_0
$\mu_0, \varepsilon_0(1 + V)$
μ_0, ε_0
z = 0
z = d

Figure 2.

Table 1

Domains of validity of the Born and the Rytov approximations in the dielectric half-space case for finite k_0 and V ($V \ll 1$)

Approximation \ Range	$z < 0$ for all z	$z > 0$ $z \ll (Vk_0)^{-1}$	$z > 0$ $z \sim (Vk_0)^{-1}$	$z > 0$ $z > (Vk_0)^{-1}$
Born	Y	Y	A	N
Rytov	N	Y	Y	Y

A: Ambiguous to estimate the scattered field or the complex phase function
N: Invalid to estimate the scattered field or the complex phase function
Y: Valid to estimate the scattered field or the complex phase function

Table 2

Domains of validity of the Born and the Rytov approximations in the dielectric-slab case for finite k_0 and V ($V \ll 1$)

Approximation \ Range	for all z $d \ll (Vk_0)^{-1}$	for all z $d \sim (Vk_0)^{-1}$	for all z $d > (Vk_0)^{-1}$
Born	Y^{-}	A	N
Rytov	Y	A	N